Complexity of Two-Phase Flow in Porous Media

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StatoilHydro
Declining oil production, increasing energy consumption: Buying time for developing of alternative energy sources.

Paul Roberts: Arguably the most serious crisis ever to face industrial society


Tar sands of Alberta, Canada

Low-permeability carbonate rock: All fluid transport through fractures - But where are the fractures, and are there enough of them?

Recovering stuck oil: EOR
Lecture Plan:

1. Transient vs. Steady-State Flow
2. Numerical Network Model
3. Steady-State Flow in Numerical Model
4. Parallel Flow Instabilities
5. Statistical Mechanics of Steady-State Flow
6. Some Thermodynamics
7. Film Flow in Crevices
8. Summary
1. Transient vs. Steady-State Flow
Defining Fluid:
- 90% by weight Glycerol - water solution
- Viscosity: 0.165 Pa s

Invading Fluid:
- Air
- Viscosity ratio: M = 0.0001
- Interfacial tension: 0.064 N/m
Flooding Experiments

Instabilities: Transients

M = 0.0001

Ca = 0.029

Ca = 0.059

Ca = 0.22

(Måløy, Univ. Oslo)

Dimensionless numbers:

- Ca = capillary number = viscous forces/ capillary forces
- M = viscosity ratio
Ca = 0.027

Ca = 0.059

Ca = 0.120
Steady State Flow

What enters a RVE is statistically the same as what leaves it.

An RVE inside a reservoir will typically experience *steady-state conditions*, and not the transients in a flooding experiment.
Steady state

signifies that apart from statistical fluctuations, the system remains constant – or changes slowly on the scale of the fluctuations.

Hence, even if both fluids move and fluid clusters break up and merge, we will be dealing with a steady state as long as the averaged macroscopic parameters describing the flow remain constant (or change slowly on the time scale of the fluctuations).

A setup for studying steady-state flow in the laboratory:
Clusters in spatially homogeneous steady-state flow

Clusters have been artificially colored.

Tallakstad, Måløy
Old wisdom:

- Equilibrium phenomena are easy
- Non-equilibrium phenomena are difficult

• Equilibrium thermodynamics is 150 years old
• Non-equilibrium thermodynamics is still being developed

Porous media:
Flooding has been studied for a long time
Steady-state flow has been largely neglected
2. Numerical Network Model
Numerical Modeling

Ca = 0.0046, M = 0.001

Ca = 0.0046, M = 100

(Aker et al., 1998)

Ca = 0.000046, M = 1

Ca = 0.000046, M = 1

(Aker et al., 1998)
Network of connected Pores

For now: no crevices, no films

Sharp corners: Wetting fluid forms films in the crevices.
Dynamics at the pore level: How do the interfaces move?
Coalescence and Instabilities: Snap-off
Coalescence and Instabilities: Coalescence
3. Steady State Flow in Numerical Model
Steady-state flow implemented on a torus

(Ramstad, 2006)
Starting from a band of red fluid, the interfaces initially develop capillary fingering and stable displacement.

$Ca = 0.000032$

$M=1$
Steady-state configuration is independent of initial preparation of system: It is a **state**.

Drainage and imbibition are now **microscopic concepts**.

(If it is independent of initial conditions given the flow is fast enough for none of the fluids to get stuck due to capillary forces.)
Reconstructed pore networks

Pore Network from Berea Sandstone

(3mm)$^3$

Each pore is described by a number of geometric parameters.

Reconstruction by e.g. merging thin slices
Setting up steady-state flow in reconstructed network:

Must make the non-periodic structure periodic
Making the three-dimensional network periodic in the flow direction:
Evolution towards steady-state flow

Steady state

Ca = 0.015, M = 1, S = 0.5

non-wetting saturation
Largest non-wetting clusters at different saturation levels in steady state

Critical saturation

Ca=0.015, M=1

S=0.59

S=0.65

S=0.67

S=0.71
Cluster size distribution corresponding to the saturations just shown.

At criticality

\[ N(s) \propto s^{-2.27 \pm 0.08} \]

Cluster size

Average over 20 samples
Relative density of largest cluster $P_\infty$ vs. saturation $S$.

Critical saturation vs. capillary number:

$$S_c = A + B \log Ca$$

Interesting consequences for radial flow
4. Parallel Flow Instabilities
Instabilities at the boundary of fluids moving in parallel.

What happens at this boundary when the capillary number is increased?

Periodic boundary conditions in the flow direction.
Kelvin-Helmholtz Shear Instability?

http://www.fluent.com/about/news/newsletters/04v13i2/a6.htm
Flow direction.

Flow periodic in the flow direction.
Mechanism:

• The non-wetting fluid produces viscous fingers that penetrate into the wetting fluid.

• The fingers experience side flow. They break off.

• This produces clusters of non-wetting fluid that diffuse into the wetting fluid and away from the non-wetting layer.

There are never any clusters of wetting fluid within the non-wetting layer.
Non-wetting saturation profile normal to flow direction as function of time.
The derivative of the non-wetting saturation profile.

This defines a *mixing length scale* $\lambda$. 
Profile is stable and moves towards the non-wetting fluid: effectively an imbibition process.
Wetting fluid

Non-wetting fluid

Mixing zone moving with constant speed

Constant current of non-wetting bubbles

Steady state in a reference frame moving with constant speed to the right.
Starting out with the wetting fluid in the middle: an equilibrium is set up.

The profile survives!
This layer is stable!
5. Statistical Mechanics of Steady-State Flow
Returning to the concept of a state.

A state is described by a small number of macroscopic parameters.

When the system is in such a state, it evolves through following a path in the space of all possible microscopic configurations such that the macroscopic averages remain constant. The equations that govern the path are the microscopic flow equations.

These equations are very hard to solve (numerically). But, they provide far too much information!

This information is needed to understand transients, but not to describe the dynamical steady state.

Is there a way to get the needed information to describe macroscopically the steady state without having to resort to complete overkill?
Sequence of configurations through time integration:

The order of the configurations has been randomized:

Does this randomization change the statistics?
If order plays no role: All steady-state properties will be completely described by the configurational probability distribution $W\{\text{cf}\}$ where $\{\text{cf}\}$ signifies the positions of all interfaces between the immiscible fluids in the porous medium.

A configuration is fully described by the position of all interfaces.

This leads to the possibility to exchange time integration by more efficient methods.
Metropolis Monte Carlo Sampling

Configurational probability

\[ W\{cf\} \]

Old configuration \( \rightarrow \) Test configuration

\[ \{cf_{old}\} \quad \{cf_{test}\} \]

Chosen by random change of old configuration.

Draw a random number \( r \in [0,1] \).

If \( W\{cf_{old}\}/W\{cf_{test}\} > r \): Reject test configuration.

If \( W\{cf_{old}\}/W\{cf_{test}\} \leq r \): Accept test configuration.
We need to know what $W_{\text{cf}}$ is?

Assumption:

\[ W_{\text{cf}} = W(E_{\text{cf}}) \]

Probability depends on single macroscopic function $E = E_{\text{cf}}$.

"Energy"
Statistical mechanics of steady-state flow

Microscopic “energy” function:

\[ E_{\text{cf}} = \frac{1}{2} \min \sum_{ij} (p_i - p_j - p_{ij}^c)^2 / \mu_{ij}^{\text{eff}} \]

Pressure field solves the Kirchhoff equations.

Effective viscosity

Capillary pressure

Temperature \( 1/T = \frac{d\Sigma}{dE} \)

\[ W_{\text{cf}} = e^{-E_{\text{cf}}/T} \]

Boltzmann distribution

Basic assumption

(All configurations with same energy are equally probable.)

Configurational entropy \( \Sigma = \log N(E) \)
All of thermodynamics now follows.

Partition function:

\[ Z = \sum \{cf\} W\{cf\} \]

Average energy (dissipation):

\[ E = \sum \{cf\} E\{cf\} W\{cf\} / Z \]

\[ E = E(\Sigma, P) \]

\[ (\partial E/\partial P)_\Sigma = -Q \]

\[ (\partial E/\partial \Sigma)_P = T \]

Thermodynamic variables:

Pressure P
Total flow rate Q
Saturation S
Fractional flow f
Energy E
Capillary pressure \( P_c \)
Entropy \( \Sigma \)
Temperature T
Viscosity ratio M
Free energy \( G = -T \log Z \)

...
Numerical model: *Controlling P (or Q) and saturation S.*

Tallakstad, Måløy, Univ. Oslo: *Controlling P (or Q) and fractional flow f.*

Monte Carlo:

\[ S = S(T, P) \]
\[ f = f(T, P) \]
6. Some Thermodynamics
Temperature:

Boltzmann weight:

\[ W_{\text{cf}} = \exp\left[-\frac{1}{2T} \sum_{ij} (p_i - p_j - p_{ij}^c)^2 / \mu_{\text{eff},ij} \right] \]

\[ W_{\text{cf}} = W(P, p^c, T) \]

Rescaling:

\[ W(P, p^c, T) = W(P/T^{1/2}, p^c/T^{1/2}, 1) \]

Setting \( T \propto P^2 \)

Equation of state

\[ W(P, p^c, T) = W(1, 1/Ca, 1) \]

Viscous pressures are all proportional to pressure \( P \)

Only dynamical parameter is Capillary number (besides \( M \)).
1st Law of Thermodynamics

\[ dE = \delta Q - \delta W \]

Reversible work related to change in flow rate \( Q \); \( P \, dQ \).

Irreversible work on the microscale related to creating new interfaces in the pores.

2nd Law of Thermodynamics

In an isolated system, a process can occur only if it increases the entropy of the system.

The steady state is at maximum entropy.
3rd Law of Thermodynamics

\[ T \to 0 \implies \Sigma \to 0 \]

The zero-temperature state, i.e., when \( P = 0 \), the state with the least number of interfaces is the optimal one. This is when the two fluids are fully separated. This state has zero entropy.
A thermodynamic equation (Not yet a complete story)

Fractional flow as a function of saturation $S$.

Pressure difference as a function of saturation $S$.

Keeping total flow rate $Q$ constant.

FIG. 2. The plots show average values of (a) fractional nonwetting flow and (b) global pressure, where the averages are taken in the steady part. The capillary number is $Ca=3.2 \times 10^{-3}$. We see clearly how these values depend on the nonwetting saturation. The simulations are run on five different geometries generated by different random seeds.
Fig. 1 – The figure shows that the relation between pressure and the derivative of the fractional is of the form in eq. (2). For these curves the viscosity ratio \( M = 1 \). Simulations with four different capillary numbers are presented for 2D, and two different capillary numbers for 3D. The nonwetting saturation \( S_{nw} \) serves as parameter for the curves, and the intervals in which the data is plotted are as follows: (2D) \( Ca = 3.2 \times 10^{-2} : S_{nw} \in (0.14, 0.93) \), \( Ca = 1.0 \times 10^{-2} : S_{nw} \in (0.14, 0.83) \), \( Ca = 3.2 \times 10^{-3} : S_{nw} \in (0.14, 0.83) \), \( Ca = 1.0 \times 10^{-3} : S_{nw} \in (0.20, 0.73) \); (3D) \( Ca = 1.0 \times 10^{-2} : S_{nw} \in (0.10, 0.90) \), \( Ca = 1.0 \times 10^{-3} : S_{nw} \in (0.20, 0.75) \).

\[
P = A \left( \frac{dF}{dS} \right)_Q + B
\]

At constant \( Q \)
Fig. 2 – The figure shows fitted curves to $P(F_{nw})$ according to eq. (3). The results are from the same series of simulations as the curves in 2D in fig. 1, only showing results for two lower capillary numbers. The shown region of $F_{nw}$ is the one where the fit is good. For these capillary numbers this region is large, for the two higher values of Ca shown in fig. 1, the region of good fit is $Ca = 3.2 \times 10^{-2} : F_{nw} \in (0.15, 0.45)$, $Ca = 1.0 \times 10^{-2} : F_{nw} \in (0.10, 0.50)$. The data is from five realizations of the porous network giving slightly shifted curves as we can see. The scattering of the data becomes larger for smaller capillary numbers due to increased hysteresis and history effects.

$$P = C F^2 + D F + E$$
This is the analytical form of the fractional flow vs. saturation curve.

This corresponds to having an analytical form for the relative permeability curves.

Fig. 6 – Data from the literature: Sharma and Yen (SY) [21], Peters and Khattan (PK) [7], Braun and Blackwell (BB) [5,22]. The water saturation is also the wetting saturation $S_w$. Different normalizations of $S_w$ in the respective papers decide the placement of the curves. The SY curves are typical samples of the set of functional forms that are employed when discussing how various physical parameters influence fractional flow. Regarding PK and BB curves, see fig. 7. All the curves are fitted to the solution of eq. (1) as is given in eq. (4).
More Phase Transitions in the Steady State  (2D system)

(Single-phase flow of wetting fluid)

(Single-phase flow non-wetting fluid)

Fig. 2. The stepwise simulation of two phase transitions. In both cases $C_0 = 1.00 \times 10^{-3}$ and $M = 10$. The saturation is changed in steps up and down past the transition. The system is run for 7 s (physical time) at each step. (a) Starting at single-phase wetting flow, increasing the saturation above the transition to two-phase flow, and then lowering the saturation back to the initial level, the system shows hysteresis. (b) Starting in the two-phase region and increasing saturation to single-phase nonwetting flow, and decreasing again, no hysteresis appears.
Three-dimensional parameter Space:

- Saturation S
- Viscosity Ratio M
- Capillary Number Ca

Both fluids move

Only one fluid moves

Fig. 3. The phase diagram for six selected values of Ca. The x-marks are the transition points from the simulations. The uncertainty in each point is not marked, but it is substantial. The scattering of points in parameter space gives an indication of the uncertainty. Phase boundaries are indicated by solid lines. See Section 4. The phase diagrams are divided into three regions, counting from the left-hand side: single-phase wetting flow, two-phase flow, and single-phase nonwetting flow.

Fig. 4. The phase diagram for three selected values of M. The simulations were performed at constant saturation, but with varying Ca. The simulated points indicate the dynamical phase boundaries. In the lower left part of the diagrams, there is single-phase wetting flow; in the middle upper part, two-phase flow; and in the lower right part, single-phase nonwetting flow.
7. Film Flow in Crevices
P: Pressure drop in bulk fluid
p: Pressure drop in film
Q: Bulk flow rate
q: Film flow rate

A: Bulk area
a: film area
The Wedge Problem in a Closed Geometry

Darcy equations for bulk and film flow:

\[ q = - \frac{k(a)}{\mu} \frac{\partial p}{\partial x} \]

\[ Q = - \frac{k(A)}{\mu M} \frac{\partial p}{\partial x} \approx \frac{k(A_T)}{\mu M} \frac{\partial p}{\partial x} \]

\[ a \ll A, \quad A_T \approx A \]

Film conservation:

\[ \frac{\partial q}{\partial x} + \frac{\partial A}{\partial t} = 0 \]

Total area:

\[ A_T = A + a \]

Total flow rule:

\[ Q_t = Q + q \]

\[ k(a) = k_0 a^2 \]
Surface Tension

Young relation

Eliminating $P$, $p$ and $q$:

\[ a = \pi \left[ \cot \frac{\phi}{2} - \frac{\phi}{2} \right] \]

\[ P - p = \frac{\gamma}{2} \]

\[ p = P - \frac{\gamma}{\pi a} \]
A Diffusion-Convection-Reaction Equation:

\[
\frac{\partial}{\partial x} \sqrt{a} \frac{\partial a}{\partial x} - \frac{\nabla^2}{A_r^2} a \frac{\partial a}{\partial x} + \frac{\nabla^5 A}{A_r^3 A} a^2 = \frac{\partial a}{\partial t}
\]

\( \nabla \times Q \)
\[
\frac{\partial}{\partial x} \left( D \frac{\partial a}{\partial x} \right) - \mu \frac{\partial a}{\partial x} + R = \frac{\partial a}{\partial t}
\]

Diffusion constant increases with increasing concentration.

\[D \propto \sqrt{a}\]

Convection speed

\[\mu \propto \frac{a}{A_T^2}\]

Reaction constant

\[R \propto \frac{A_T^{1/2}}{A_T^3} a^2\]

Shock fronts
The area along the tube is constant:

\[ A'_x = 0 \implies \]

\[ \frac{\partial}{\partial x} a^\beta \frac{\partial a}{\partial x} - a \frac{\partial a}{\partial x} = \frac{\partial a}{\partial t} \]

\[ \beta = 0 \implies \text{Burgers' equation} \]

\[ \beta = \frac{1}{2} \implies \text{Film equation} \]

When \( Q = 0 \): No bulk flow.

Convection term is zero.
No bulk flow: Diffusion only

\[
\frac{\partial}{\partial x} \sqrt{\alpha} \frac{\partial \alpha}{\partial x} = \frac{\partial \alpha}{\partial t}
\]

\[y = \frac{x^2}{t}\]

\[
\exp\left[\left(y - 2y_0\right)/\alpha_0^{\frac{1}{2}}\right] = e^{2\sqrt{\alpha}} e^{\tilde{y} - 2\tilde{y}_0}
\]

\[
\frac{1 - \sqrt{\alpha}}{1 + \sqrt{\alpha}} e^{2\sqrt{\alpha}} = e^{\tilde{y} - 2\tilde{y}_0}
\]

\[
e^{-\arctan \sqrt{\alpha} + 2\sqrt{\alpha}} = e^{\tilde{y} - 2\tilde{y}_0}
\]

\[
\tilde{\alpha} = \frac{\alpha}{|\alpha_0|} \quad \text{and} \quad \tilde{y} = y / \sqrt{|\alpha_0|}
\]
Solutions:
Shock front

Physical solution
Bulk flow: Non-linear diffusion-convection equation

\[ \frac{\partial}{\partial x} \sqrt{a} \frac{\partial a}{\partial x} - \frac{\nu}{A_r^2} a \frac{\partial a}{\partial x} = \frac{\partial a}{\partial t} \]

Solitary wave solution

\[ \sqrt{\alpha_+} \log \left| \frac{1 + \sqrt{\alpha/\alpha_+}}{1 - \sqrt{\alpha/\alpha_+}} \right| + \sqrt{\alpha_-} \log \left| \frac{\sqrt{\alpha/\alpha_-} - 1}{\sqrt{\alpha/\alpha_-} + 1} \right| = \frac{4\nu a_1}{wA_r^2} (z - 2z_0) \]

Integration constants

\[ Z = x - u_t \]

Solitary wave speed
Solitary wave speed \( u \propto (a_+ + a_-) \)
Solitary wave speed \( u \propto (a_+ + a_-) \)
Stationary solutions:

\[ \frac{\partial}{\partial x} \sqrt{a} \frac{\partial a}{\partial x} = \frac{\nu}{A_f^2} a \frac{\partial a}{\partial x} \]

\[ \log \left[ \frac{1 + \sqrt{\kappa \alpha_2}}{1 + \sqrt{\kappa \alpha_1}} \right] - \log \left[ \frac{1 - \sqrt{\kappa \alpha_2}}{1 - \sqrt{\kappa \alpha_1}} \right] 
- 2 \arctan \left[ \frac{\sqrt{\kappa \alpha_2} - \sqrt{\kappa \alpha_1}}{1 + \kappa^2 \alpha_1 \alpha_2} \right] = 2\kappa^{3/2} C |(\xi_1 - \xi_2)|. \]
Solvable under steady-state conditions for a class of profiles

\[ \frac{\partial}{\partial x} \sqrt{A} \frac{\partial a}{\partial x} - \frac{\nu}{A_t^2} a \frac{\partial a}{\partial x} + \frac{\nu}{A_t^3} A_t a^2 = \frac{\partial a}{\partial t} \]

\[ \nu \propto Q \]

\[ A_t = A_t(x) \]

\[ A_t^{3/2} = A_0^{3/2} + C(x-x_0) \]
Asymptotic solution:

\[
\frac{\alpha}{\alpha_i} = \frac{A}{A_i} \left[ 1 - \left( \frac{\alpha_i}{A_i} \right)^2 \left( \frac{3v}{8w} \right) \left( \frac{2w|C|}{v} \right)^{1/4} \log \left( \frac{A}{A_i} \right) \right]^{-1/2}
\]

There is a divergence for finite \(A_{\tau}/A_i\) for which \(a\) diverges.

Critical area at which film goes unstable.

\[
A_c = A_i \left( \frac{4\pi L Q M \mu}{\sqrt{\lambda}} \right)^{5/2} \left( \frac{A_i}{\alpha_i} \right)^{5/2}
\]
This instability leads to bubble formation in the pores.
\[ A_T = 0.1 + (x + 0.5)^2 \]
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